EXPLAINING FRACTAL DIMENSION IN POPULOUS CITIES

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Abstract

Cities are transforming at varying levels from local to global. Alternative ways of measuring this transformation have long been applied, and fractal dimension is one of them. A number of scholars have been studying the urban structure through the usage of fractals and have assessed the relationship between fractal structures and built-up urban areas. Fractal dimension is considered as a statistical magnitude measuring space-filling efficiency, where the space-filling efficiency increases with fractal dimension. However, the scope of the studies using fractal dimension as a measure of space-filling efficiency in cities have been limited to deriving fractal dimensions for either cities or parts of the cities, such as neighborhoods. None of the past studies have aimed to explain the level of fractal dimension pertaining to cities using economic, social and demographic variables. This study aims to construct a simple statistical model to explain the variation in the space-filling efficiency of cities using economic, social and demographic explanatory variables. The space-filling efficiency at the city level is measured by the fractal dimension calculated using box-counting method. The results from a set of randomly selected 17 cities show that the variation in space-filling efficiency of worldwide cities can well be explained by Human Development Index (HDI), population change, income per capita, number of vehicles per capita, and fuel price. The findings indicate that these variables can be used to control the level of space-filling efficiency and develop policies regarding to urban growth possibly against urban sprawl.

Key Words: Fractal Dimension, Space-filling efficiency, Urban Growth.

1. INTRODUCTION

The term fractal is used to indicate a rough or fragmented geometric shape, introduced by Benoit

Mandelbrot in 1977, to describe geometric figures that exhibit self-similarity. Natural spatial objects, including coastlines, plants and clouds have long been treated as fractals of various dimensions (Mendelbrot 1983). Mathematically, the fractals have no relation with scales. A city can be treated empirically as a fractal system with self-similarity (Chen, 2013).

Concept of fractal dimension has already been used in both of physical and human geography (Mandelbrot, 1983). Fractals are spatial objects whose geometric characteristics include scale dependence, irregularity, and self-similarity (Shen, 2002). In the recent years, many researchers have tried to understand the dynamics of urban land change and its complex impact on systems using fractals.

Recent research has demonstrated that the urban form cannot be fully described by Euclidean geometry, but rather be treated as fractals (Batty and Longley, 1987; Benguigui, 1991; Batty and Xie, 1996; 1999; Shen 1997; 2002). Ever since the very first introduction of fractals by Mandelbrot in 1977, many scholars have studied the urban form, or urban structure, through the usage of fractals. Numerous empirical studies provide evidence that the relationship with urban growth and fractal dimension (Benguigui, 1991; De Keermaecker ve diğer., 2003; Frankhauser, 1998). Urban form has been empirically demonstrated that it can be described using fractal geometry (Chen, 2013).

The fractal dimension values of urban patterns were discussed by Thomas and others (2008) and statistical relationship between land use patterns. Spatial correlation analysis known as a useful tool for urban studies (Chen, 2010; 2013). In general; urban growth is explained demographic, socioeconomic and physical factors. According to the literature prove different kinds of push and pull factors motivate urban growth.

With this background; this paper aims to analyze the structure of cities by fractal dimension. The fractal dimension of cities is explained by the variables commonly used in spatial urban economics, socio-cultural structure and urban planning through regression. The sample includes 17 randomly selected cities. The remaining of the paper is organized as follows. Section 2 introduces methodology and data; section 3 presents the analysis and results. Conclusions and perspectives are included in Section 4.

2. METHODLOGY AND DATA

Fractal cities are generally defined in a 2-dimension space based on a remotely sensed image or a digital map (Chen, 2013). Several methods are used for urbanized areas. It is argued in the literature that using box-counting method is the most appropriate method in deriving fractal dimension, when the built-up areas are in question. Thus, following, Benguigui and Daoud, 2001; Shen, 2002; De Keersmaecker et al., 2003; Kaya and Bölen, 2006; Mcadams, 2007; Sun et al., 2007; Terzi and Kaya, 2008; Yanyan et al., 2008; Yi et al., 2008; Cubukcu and Cubukcu, 2009; Feng and Chen, 2010; Thomas et al. ,2008; Thomas et al., 2010; Chen, 2011; Tannier et al., 2011; Atabeyoğlu and Bulut, 2013 he box counting method is selected to calculate the fractal dimension of cities.

In the box-counting method; grids in varying sizes are overlaid on a 2-D the black and white image of built-up areas, and the number of black cells are counted for each and every grid size. The black cells represent the built-up areas and the white ones un-built areas. The logarithm of *N(r),* the number of occupied cells versus the logarithm of *1/r* (r is the size of cell) gives a line correspond to the box dimension. The fractal dimensions are the least-square estimates of their true fractal dimensions:

$$\log(n(s)) = \log(U) + D\mathrm{Log}(1/s) + \varepsilon_s,$$

where, log(U) is the constant with U being the built-up urban area size, ε_s is the error term, and D is the estimated fractal dimension. The fractal dimension is the slope between log(n(s)) and log(1/s) (Shen, 2002)(Figure 2.1) :

$$\log(n(s))= \log(U) + D\log(1/s) + \varepsilon s$$

D= fractal dimension
Log(U) = constant (U is built area)
s= box size
εs= error term.

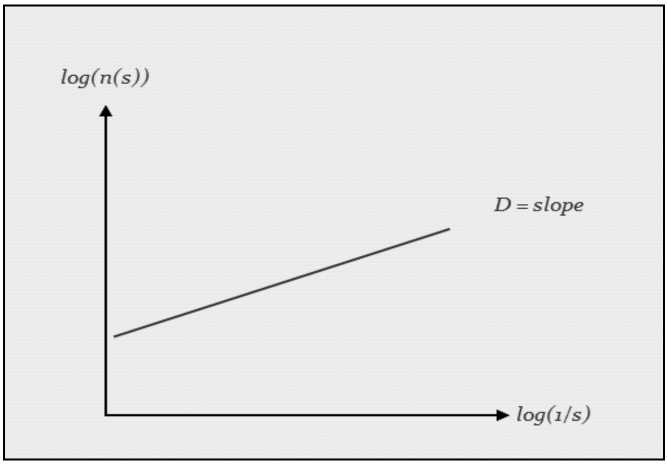


Figure 2.1 The slope is the fractal dimension

Fractal dimension is a quantitative measure of the efficiency of space-filling. It is a real number, often between 1 and 2, which implies that fractal objects occupy irregularly shaped spaces (Ball, 2004). Fractal dimensions are used as indicators of complexity of curves and surfaces.

This study examines the urban space-filling efficiency measured by fractal dimension pertaining to randomly selected 17 high populated world cities from most populated 500 cities list using urban explanatory variables (Table 2.1 and Table 2.2).

Table 2.1 Selected cities and continental information

NO	**CONTINENTAL**	**COUNTRY**	**CITY NAME**

1	AMERICA	MEXICO	MEXICO CITY
2	ASIA	THAILAND	BANKOGK
3	EUROPE	TURKEY	ISTANBUL
4	ASIA	CHINA	TIENTSIN
5	AMERICA	USA	CHICAGO
6	ASIA	JAPAN	NAGOYA
7	ASIA	MALESIA	KUALA LUMPUR
8	AMERICA	CHILI	SANTIAGO
9	ASIA	RUSSIA	SAINT PETERSBURG
10	EUROPE	SPAIN	BARCELONA
11	ASIA	JAPAN	TOKYO
12	EUROPE	NORWAY	OSLO
13	ASIA	JAPAN	YOKOHOMA
14	ASIA	JAPAN	KOBE
15	ASIA	SOUTH KOREA	DOEJEAN
16	ASIA	SOUTH KOREA	SEOUL
17	ASIA	PHİLİPPİNES	QUEZON CITY

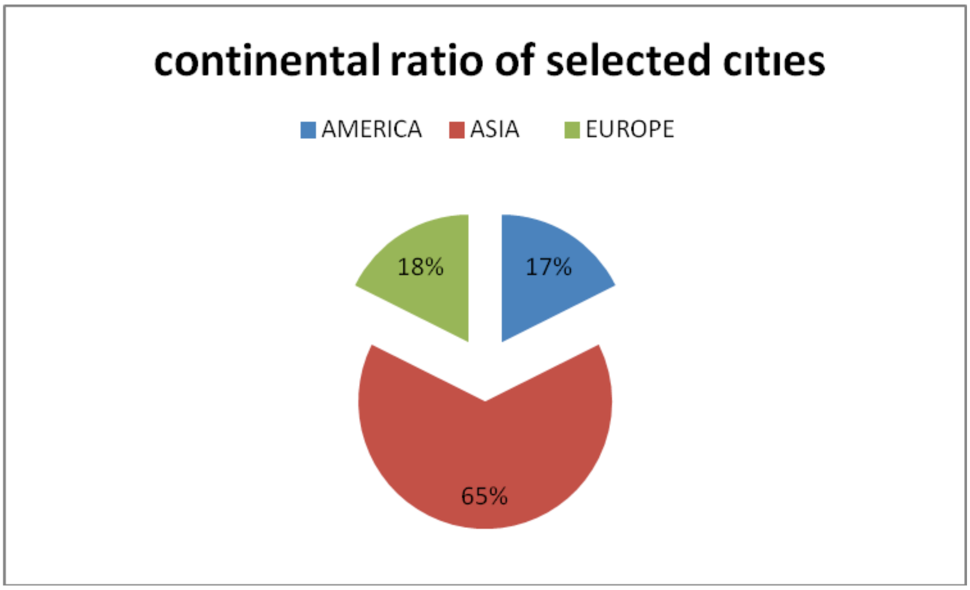


Figure 2.2 Continental Ratio of Selected Cities

The 65% of the cities in the sample are Asian cities, 17% American cities, and 17% European cities (Figure 2.2). This distribution pertaining to the sample is very close to the population, which is the whole world (Figure 2.3). The sample has no cities from Africa and Oceania. That is plausible since the sample is chosen from the most populated 500 cities.

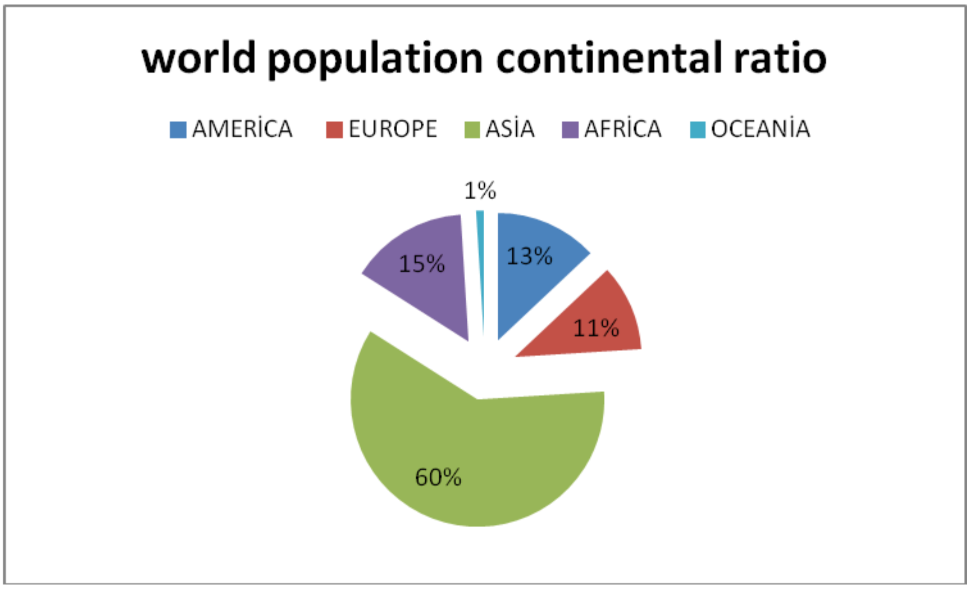


Figure 2.3 World Population Continental Ratio

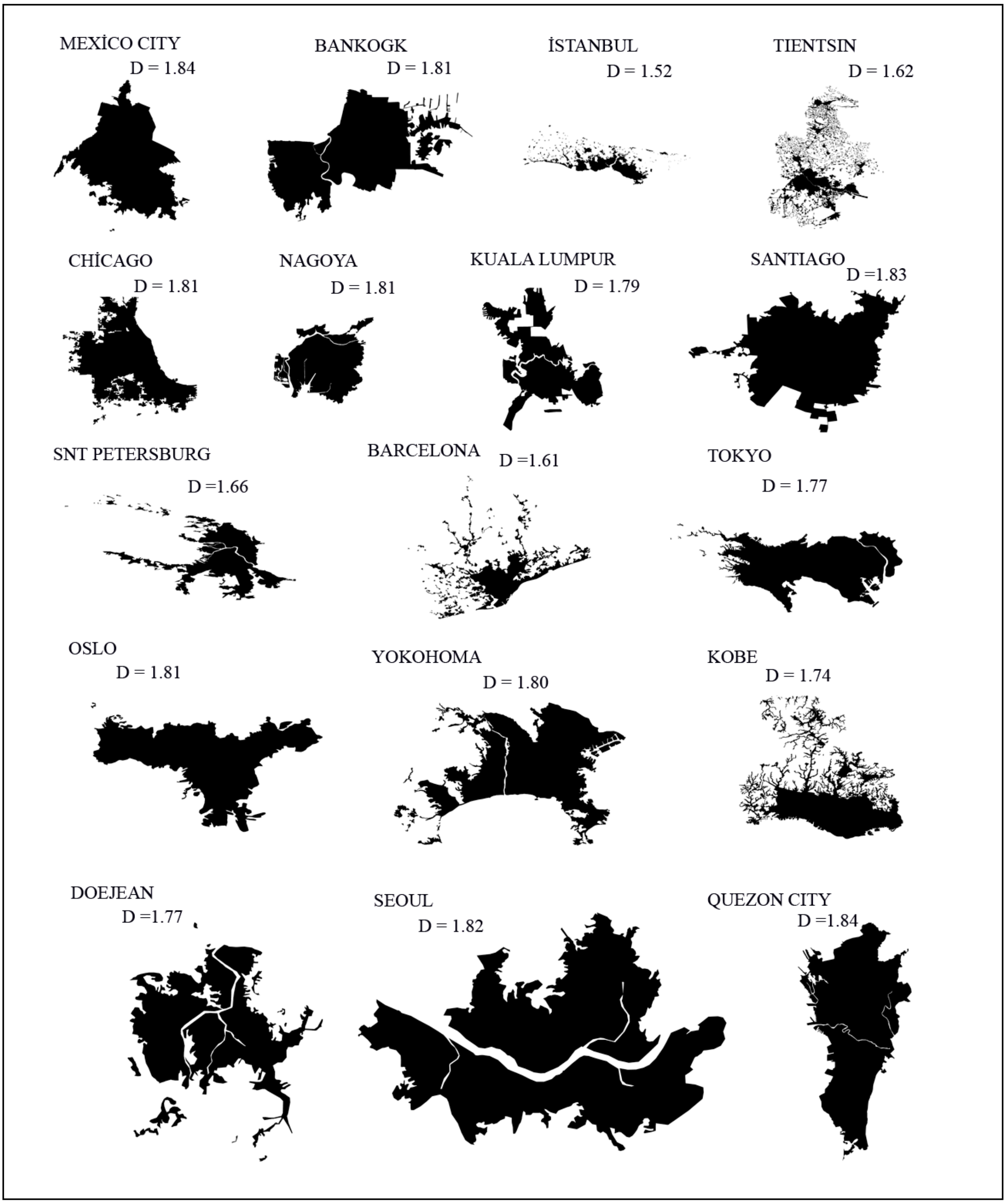
The basic input for calculating the fractal dimensions is the satellite images of the cities in the sample showing the built-up areas. The images of the 17 urbanized areas are digitized from the digital images provided by GoogleEarth. The urban forms were then scaled, projected, and formatted using geographic information systems. The fractal dimensions are calculated using, Fractalyse, a fractal analysis software developed by the research team "mobilities, city and transport" of the research center ThéMA at Université de Franche-Comté. The fractal dimensions of the cities in the sample are shown in Table 2.2. The fractal dimension values for the sample varies between 1.52 (Istanbul) and 1.84 (Mexico City and Quenzon City).

Table 2.2 Fractal Dimension Estimates for 17 Cities Settlement Area and Population

NO	COUNTRY	CITY NAME	POPULATION RANK OF THE CITY	POPULATION	FRACTAL DIMENSION
1	MEXICO	MEXICO CITY	9	23,200,000	1.84
2	THAILAND	BANKOGK	28	13,700,000	1.81
3	TURKEY	ISTANBUL	20	13,300,000	1.52
4	CHINA	TIENTSIN	26	9,800,000	1.62
5	USA	CHICAGO	19	9,750,000	1.81
6	JAPAN	NAGOYA	18	2,266,249	1.81
7	MALESIA	KUALA LUMPUR	30	6,450,000	1.79
8	CHILI	SANTIAGO	47	6,100,000	1.83
9	RUSSIA	ST. PETERSBURG	35	5,050,000	1.66
10	SPAIN	BARCELONA	73	4,575,000	1.61
11	JAPAN	TOKYO	1	13,159,388	1.77
12	NORWAY	OSLO	375	1,502,604	1.81
13	JAPAN	YOKOHOMA	172	3,697,894	1.80
14	JAPAN	KOBE	235	1,533,852	1.74
15	SOUTH KOREA	DOEJEAN	247	1,476,736	1.77
16	SOUTH KOREA	SEOUL	4	10,020,123	1.82
17	PHİLİPPİNES	QUEZON CITY	184	2,173,831	1.84

The size and complexity of urbanized areas are influenced by many other city parameters. Thus, fractal dimensions may be used as an important parameter for urban form and growth modeling (Shen, 2002). As also suggested in Shen (2002), this study indicates that the cities with the same fractal dimension values and urbanized areas may have quite different population sizes examples as; Bankogk, Chicago, Nagoya, Oslo (1.81); Mexico City and Quenzon City (1.84); Tokyo and Doejean (1.77) (Figure 2.4). This finding shows that the fractal dimension alone is an indicator of the total of urbanized areas but not a good measure of urban density (Shen, 2002). In regards, this study aims to explain the fractal dimension of cities with demographic, socioeconomic and physical factors using a regression model.

Figure 2.4 Urbanized Areas of 17 Cities with Fractal Dimensions



The dependent variable in the model is the fractal dimension, measured by the box-counting method. The set of explanatory variables tested in the regression model include:

(1) Population change, gross national product in 2009, vehicle number per 1000 people, pump price for gasoline, total employment, urban agglomeration index (available from the World Bank database),

(2) HDI index (available from the Human Development Report, published by the United Nations),

(3) Total country area (available from worldatlas.com),

(4) Latitude data (available from the official web site of the cities).

The dependent variable, fractal dimension calculated using box-counting method in exponential linear for is in the natural logarithmic form (ln). The explanatory variables total labor force and pump price gasoline are used in square forms (sq), and vehicle number and population urban agglomeration in in natural logarithmic forms (ln).

3. ANALYSIS AND RESULTS

The selected model is a log-log linear model. The depended variable is the linear exponential fractal dimension. The explanatory (independent) variables in the selected model include: (1) total country area, (2) gross national product in 2009, (3) HDI index, (4) vehicle number per 1000 people (log mod), (5) population agglomeration (log mod), (6) pump price for gasoline (square mod), total employment in the country (square mod), population change and latitude. The selected model explains % 87 of the variance in fractal dimension of the selected sample (R^2=0.875, adjusted R^2=0.714).

Table 2.4 presents the parameter estimates for the selected model. The fractal dimension increases when latitude, gross national product, HDI index increase. On the other hand, it decreases when the total country area, population change, pump price for gasoline, total labor force, urban agglomeration ratio, and the vehicle number per 1000 people decrease. All the variables but latitude are statistically significant at the .05 level. The latitude variable is not statistically significant at the 0.05, it is found to be contributing to the model statistically. The variance inflation factors (VIF) calculated for each variable in the selected model show no signs of multicollinearity.

4. CONCLUSIONS AND PERSPECTIVE

Cities are complex organism variation of economical, social and technological and generally be limited by natural environment. Urbanized areas can be geometrically planned and designed using Euclidean geometry on the other hand urban form and its systems cannot be described by Euclidean geometry. The quantitative research on urban form is very helpful in understanding urban spatial morphology. Fractal dimension is one of quantitative methods can be calculated for urbanized areas. Various scientific disciplines have been used fractal dimensions for more than five decades. Fractals preferred by scholars for two main advantages. First, storing data pertaining to urban boundaries at different scales is time and money consuming. Fractal dimension can avoid disadvantage of scale. Second, Euclidean geometry is not satisfying to explain complex spatial forms.

Table 2.4 Parameter Estimations*

Variable	Estimated Coefficients Unstandardized	Standard Error	t	p-value (Pr > | t |)	**VIF**
Total country area	-1.671 * 10-4	0.000	-4.306	0.004	0.151
GNP 2009	1.291 * 10-4	0.000	2.557	0.038	0.711
HDI index	,759	0.207	2.553	0.038	0.457
Vehicle number (ln)	- .252	0.078	-3.209	0.015	0.771
Population urban agglomeration (ln)	- .172	0.045	-3.781	0.007	0.350
Total labor force (sq)	-3.600 * 10-6	0.000	-3.033	0.019	0.245
Pump price gasoline (sq)	- .047	0.010	-4.601	0.002	0.008
Population change	- .102	0.035	-2.945	0.022	0.767
Latitude	4.796 * 10-6	0.000	1.773	0.119	0.346

* : Dependent Variable: Fractal Dimension (LN)

In this study, the box-counting fractal dimension algorithm were presented and discussed. The objective of this paper was explained urban growth with fractal based parameters with linear regression methods. Urban built-up areas of 17 cities to explain spatial variations by means of variables commonly used in geography, urban planning and urban economics.

Our findings suggest that the total country area, population change, pump price for gasoline, total labour force, urban agglomeration ratio, vehicle number per 1000 people, latitude and gross national product rates are statistically significant estimators of urban growth.

Cities with the same fractal dimension values and urbanized areas may have quite different population. As also shown in Shen (2002), this observation implies that fractal dimension cannot be explained by only urban density. This study confirms that fractal dimension can be explained by social, economic and demographic variables, and leads to the conclusion that the urban pattern is an outcome of social and economic activities in the city. The findings indicate that these variables can be used to control the level of space-filling efficiency and develop policies regarding to urban growth possibly against urban sprawl. Further research may consider a wider spectrum of local social and economic variables.

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